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ANISOTROPIC MATERIALS

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ISOTROPIC ABAQUS UEL- COMPLEX TAYLOR SERIES METHOD (CTSE)

The beginning of the semester starting in January 2013, the continued work for the numerical example in the manuscript submission “Improved WCTSE method for the generation of 2D weight functions through implementation into a commercial finite element `code`” was completed.

Comment [HRM1]: Revise?

In conjunction and continued following that was the research on anisotropic materials, learning basic FORTRAN programming, and programming a 2-D isotropic User Element (UEL) in Abqauqs to implement the Complex Taylor Series Method (CTSE) to the built-in Abaqus elements.

Upon completing programming of a 3-D Isotropic element implementing CTSE will be continued work throughout the semester.

ANISOTROPIC MATERIALS

Anisotropic Materials

Anisotropic materials are materials whose properties are directionally dependent. Unlike isotropic materials that have material properties identical in all directions, anisotropic material’s properties such as Young’s Modulus, change with direction along the object. Common examples of anisotropic materials are wood and composites.

Implications

Directionally dependent physical properties of anisotropic materials are significant due to the affects it has on how the material behaves. For example, in the case of fracture mechanics, the way the microstructure of the material is oriented will affect the strength

and stiffness of the material in various directions therefore affecting direction of crack growth.

Applications

Anisotropic materials, naturally and man-made, are used in multiple areas of study. Some examples are Magnetic anisotropy in which the magnetic field is oriented in a preferred direction, anisotropic heat conduction that is dependent on the geometry and or anisotropic material. Anisotropic materials are also a result of manufacturing of materials such as a rolling or deep-drawing process. Composites and other materials are used and altered for specific applications.

Theory

Hooke's Law

Based on Linear Elasticity, stress and strain of a material is related by the constitutive, or “stiffness”, matrix. This relationship is defined by the equations and matrices below using Hooke's Law:

$$\sigma_{ij} = C_{ij}\epsilon_{ij}$$

$$\epsilon_{ij} = S_{ij}\sigma_{ij}$$

Where σ_{ij} and ϵ_{ij} describe the stress and strain components, i indicating direction of normal to plane and j indicating direction of component. C_{ij} is the material's stiffness or Elastic Constant and S_{ij} is the compliance or inverse of C_{ij} .

The stiffness tensor is a fourth order tensor C_{ijkl} that is originally consisting of 89 components. The Compliance matrix is 1/stiffness and will follow the same concepts to

be described. However due to symmetry $ijkl=jikl$ and $ijkl=ijlk$ and other relationships as shown in the reduced matrix below, simplifies to 36 components.

The relationship of shear components $ij=ji$ reduces this further. By this concept one can then see that the 6 diagonal components are related to the normal stress and strains: $36-6=30$. The remaining 30 can be divided by two with the shear relationship $30/2=15$ independent components. Thus the 6+15 components of the matrix results to 21 independent components of the stiffness/compliance matrix for anisotropic materials.

1111	1112	1113	1121	1122	1123	1131	1132	1133
1211	1212	1213	1221	1222	1223	1231	1232	1233
1311	1312	1313	1321	1322	1323	1331	1332	1333
2111	2112	2113	2121	2122	2123	2131	2132	2133
2211	2212	2213	2221	2222	2223	2231	2232	2233
2311	2312	2313	2321	2322	2323	2331	2332	2333
3111	3112	3113	3121	3122	3123	3131	3132	3133
3211	3212	3213	3221	3222	3223	3231	3232	3233
3311	3312	3313	3321	3322	3323	3331	3332	3333

$iikk$ $9/1 = 9$ $ijkk$ or $iikl$ $36/2 = 18$
 \rightarrow $ijkl$ $jikl$ $ijlk$ or $jilk$ $36/4 = 9$
 $9 + 18 + 9 = 36$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} C_{11} & & & & & \\ & C_{22} & & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{13} \\ 2\epsilon_{23} \end{Bmatrix}$$

symm.

Material Rotation

Crystallography

The stress and strain of the material are affected by the rotation of the material matrix with respect to the global coordinate system of the object. The effects are dependent on the particular materials' crystallographic characteristics; it is as this micro-level structure that dictates the materials' behavior.

Euler Angles

The rotation of the material matrix is done by implementing Euler Angles using Bunge (ZXZ) notation is the method selected as the rotation matrix transformation for the stiffness matrix, stress, and strain components. This notation is as the name describes, the first angle of rotation is about the z-axis ϕ_1 , then the x-axis Φ , and the z-axis again ϕ_2 . The rotation matrices for each axis are listed below in equations R_x and R_y , respectively. Lastly the resulting matrix is listed in equation R.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_1 = \cos(\phi_1) \quad s_1 = \sin(\phi_1)$$

$$c_2 = \cos(\phi_2) \quad s_2 = \sin(\phi_2)$$

$$C = \cos(\Phi) \quad S = \sin(\Phi)$$

$$R = \begin{bmatrix} c_1 c_2 - s_1 s_2 C & s_1 c_2 + c_1 s_2 C & s_2 S \\ -c_1 s_2 - s_1 c_2 C & -s_1 s_2 + c_1 c_2 C & c_2 S \\ s_1 S & -c_1 S & C \end{bmatrix}$$

This rotation matrix is used to relate the local orientation of the material to the global coordinate system of the object as a whole. The coordinates of the nodes can be calculated by the following example:

$$x_{\text{local}} = R x_{\text{global}} \rightarrow x_{\text{global}} = R^{-1} x_{\text{local}}$$

This same concept is repeated for the y and z coordinates.

The rotation matrix is also applied to the matrices in Hooke's Law to see how the stresses change depending on the orientation of the material.

Abaqus

The process of solving these problems is using input files containing the nodes, elements, grains, and material orientation all being called into one input file. Anisotropic materials are broken up into element sets called grains, and each of these grains is oriented in different directions, resembling an actual material. Each element consists of nodes, in this case the built-in Abaqus element TYPE=C3D8 which is a 3-D 8-noded linear isoparametric element. The first example used is showing the effect of rotating the material matrix of a single element and grain.

In order to rotate the local element with respect to the global coordinate system of the element, the rotation matrix R as described above was used to transform the x, y, and z coordinates using the ORIENTATION function in Abaqus. The transformed coordinates are put into the orientation input file.

The application of the CTSE method to test the sensitivities of certain material properties and/or orientation angle are the next steps that will be taken and tested. This will be accomplished by programming a User Element (UEL) in Fortran to use in

Abaqus. A comparison of CTSE results with traditional methods will be compared in a numerical example